

Supersymmetry, Naturalness, and Light Higgsinos*

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Abstract

We compare and contrast three different sensitivity measures, Δ_{EW}^{-1} , Δ_{HS}^{-1} and Δ_{BG}^{-1} that have been used in discussions of fine-tuning. We argue that though not a fine-tuning measure, Δ_{EW} , which is essentially determined by the particle spectrum, is important because Δ_{EW}^{-1} quantifies the *minimum* fine-tuning present in any theory with a specified spectrum. We emphasize the critical role of incorporating correlations between various model parameters in discussions of fine-tuning. We provide toy examples to show that if we can find high scale theories with specific correlations amongst parameters, the value of the traditional fine-tuning measure Δ_{BG}^{-1} (which differs significantly from Δ_{HS}^{-1} only when these correlations are important) would be close to Δ_{EW}^{-1} . We then set up the radiatively driven natural SUSY framework that we advocate for phenomenological analyses of natural models of supersymmetry, and review the implications of naturalness for LHC and ILC searches for SUSY as well as for searches for SUSY dark matter.

Keywords: supersymmetry, naturalness, higgsino signatures, Large Hadron Collider, Linear Collider

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1 Introduction

It is common knowledge that effective theories valid above some distance scale provide an excellent description of phenomena down to that scale. Hydrodynamics does not require us to even know about the existence of atoms, and applies at distance scales much larger than the size of atoms. Likewise, an understanding of the atoms and molecules does not require knowledge of quarks or even of nuclear physics, for that matter. The reductionist's hope is that *the principles* governing all phenomena stem from a fundamental underlying theory which, in turn, would enable us to derive seemingly fundamental concepts from a deeper origin. The derivation of the empirical laws governing the behaviour of ideal gases from kinetic theory provides a simple illustration of this. A different example is the derivation of the Stefan-Boltzmann law for the emissive power of a blackbody (which also enables us to write Stefan's constant in terms of the more fundamental Planck's constant). The derivation of the magnetic susceptibility and polarizability of mono-valent gases in terms of atomic properties of the corresponding atoms provides an illustration of how “long-distance characteristics” – in this example, some bulk properties of gases – can be obtained from the underlying microphysics. Continuing in this vein, we may hope that in the future, some of the many disparate parameters of the Standard Model which has been remarkably successful in describing data up to distance scales down to $(100 \text{ GeV})^{-1}$, will be derived from an underlying (more) fundamental theory that includes a detailed description of new degrees of freedom with mass scales (much) higher than 100 GeV.

It is perhaps worth emphasizing that realizing such a top-down program may prove very difficult, even in principle, because the low energy theory may turn out to be sensitive to physics at all energy scales. Although most of us implicitly assume that very high energy scale degrees of freedom decouple from low energy physics, it remains logically possible that this may not be the case. It could, for instance, be that the multiplicity of massive states grows so rapidly with mass, that even though the effect of any individual state is negligible, their collective effect remains at low energy. In this case, one would have to know the detailed physics at all energy scales to realize the top-down program.

The other possibility is that low energy physics is insensitive to the details of high scale physics because the effects of the latter are suppressed by a power of the high scale Λ . This view provides a rationale for the success that renormalizable relativistic quantum field theories have enjoyed in the describing strong and electro-weak phenomena today, and makes a strong case that any mass scale associated with unknown degrees of freedom lies well above the highest energies accessible today, so that the effect of non-renormalizable operators is sufficiently suppressed.

There is, however, an associated issue, brought to the forefront by the discovery of the first (seemingly) elementary spin-zero particle at the CERN collider with attributes remarkably consistent with those of the Higgs boson of the Standard Model [1, 2]. Although, as we just said, low energy phenomena are essentially independent of Λ , the *dimensionful* parameters of the *renormalized* theory are generally speaking sensitive to the high scale Λ , and hence to the physics at high energy scales. For instance, in a generic quantum field theory, the radiative corrections to the squared mass of an elementary spin-zero particle take the form,

$$m_\phi^2 - m_{\phi 0}^2 = C_1 \frac{g^2}{16\pi^2} \Lambda^2 + C_2 \frac{g^2}{16\pi^2} m_{\text{low}}^2 \log \left(\frac{\Lambda^2}{m_{\text{low}}^2} \right) + C_3 \frac{g^2}{16\pi^2} m_{\text{low}}^2 . \quad (1)$$

The C_3 term could also include “small logarithms” $\log(m_{\text{low}}^2/m_\phi^2)$ that we have not exhibited. We see the well-known quadratic sensitivity of scalar mass parameters to the scale Λ where new massive degrees of freedom that couple to the SM reside; *e.g.* $\Lambda = M_{\text{GUT}}$ when the SM is embedded in a Grand Unified framework.¹ In Eq. (1), m_ϕ is the physical mass of the quantum of the field ϕ , g is the typical coupling of the field ϕ , $m_{\phi 0}$ is the corresponding mass parameter in the Lagrangian, $16\pi^2$ is a loop factor, and C_i are dimensionless coefficients that aside from spin, colour and other multiplicity factors are numbers $\mathcal{O}(1)$. Finally, m_{low} denotes the highest mass scale in the low energy theory, while Λ is the scale at which this effective theory description becomes invalid because the effects of heavy states not included in the Lagrangian that provides a description of physics at low energies become important. If $\Lambda \gg m_{\text{low}}$, unless g is also tiny, the first term dominates the corrections. Moreover, in order for the physical mass m_ϕ to be at its fixed value in the low energy theory, it must be that there are large cancellations between $m_{\phi 0}^2$ and the Λ^2 term in Eq. (1). This quadratic sensitivity of the radiative corrections to the squared mass parameter of elementary spin-zero fields leads to the *fine-tuning problem* in the Standard Model (SM) [5] when the SM is embedded into a Grand Unified Theory. We stress that this is not a logical problem in the sense it does not render the theory inconsistent, nor a practical problem that precludes the possibility of making precise predictions using the SM. It is only a problem in the sense that seemingly unrelated quantities in Eq. (1) — the mass parameter $m_{\phi 0}^2$ of the low energy Lagrangian and contributions from radiative corrections from very massive degrees of freedom governed by very different physics — need a cancellation of many orders of magnitude if $\Lambda \sim M_{\text{GUT}}$. Why should two quantities with very different physical origins balance out with such exquisite precision?

The remarkable ultra-violet properties of softly broken supersymmetric (SUSY) theories, with SUSY broken near the weak scale, ensure that the low energy theory is at most logarithmically sensitive to high scale (HS) physics, *i.e.*, that the C_1 term in Eq. (1) is absent. This led to the realization [6] that weak scale SUSY potentially solves the *big gauge hierarchy problem* endemic to the Standard Model (SM) [5] embedded into a GUT framework, and provided much impetus for its study over the last three decades. The recent discovery of a Standard Model (SM)-like Higgs boson with mass $m_h \simeq 125 - 126$ GeV [1, 2] at the LHC seemingly provides support for the simplest SUSY models of particle physics [7, 8] which had predicted $m_h \sim 115 - 135$ GeV [9]. However, no sign of supersymmetric matter has yet been found at the LHC, resulting in mass limits $m_{\tilde{g}} \gtrsim 1.5$ TeV (for $m_{\tilde{g}} \simeq m_{\tilde{q}}$) and $m_{\tilde{q}} \gtrsim 1$ TeV (for $m_{\tilde{g}} \ll m_{\tilde{q}}$) [10, 11]. Naively, this pushes up the SUSY scale m_{low} to beyond the TeV range. If Λ is not much above the SUSY scale, the $C_{2,3}$ terms in Eq. (1) each have a scale $\sim (100 \text{ GeV})^2$, which is comparable to the observed value of the Higgs boson mass for $m_{\text{low}} \lesssim 1 - 2$ TeV, and no large cancellations are necessary. However, one of the most attractive features of supersymmetric theories is that they can be perturbatively valid up to energy scales as high as M_{GUT} at which the measured values of the three SM gauge couplings appear to unify. In this case,

¹We stress that Λ here is not a *regulator* associated with divergences that occur in loop calculations in quantum field theory. Rather, it is the mass scale associated with new particles with large couplings to the Higgs boson, a point also made explicitly in Ref. [3]. From this viewpoint, and tempting though it is, we would not logically be able to associate Λ with M_{Planck} , the scale at which the effects of gravity become important. We do not really know quantum gravitational dynamics and, in particular, do not know that there are associated new particles with significant couplings to the Higgs boson. See also Ref. [4].

$\Lambda \sim M_{\text{GUT}} \simeq 2 \times 10^{16}$ GeV, so that the C_2 term becomes two orders of magnitude larger than $(100 \text{ GeV})^2$, requiring cancellations at the percent level to obtain measured value of the Higgs boson mass. This need for fine-tuning is what has been termed as the *Little Hierarchy Problem*, to be contrasted with the *Big Hierarchy Problem* that is solved by the introduction of weak scale SUSY as we mentioned earlier.

Fine-tuning in the Minimal Supersymmetric Standard Model (MSSM) is seemingly exacerbated because experiments at the LHC have discovered a SM-like Higgs boson with a mass at 125-126 GeV, well beyond its tree-level upper bound $m_h \leq M_Z$. Radiative corrections can readily accommodate this, but only with top squark masses beyond the TeV scale along with large mixing[12]. Since top squarks have large Yukawa couplings to the Higgs boson, it has been argued that naturalness considerations prefer $m_{\tilde{t}_{1,2}}, m_{\tilde{b}_1} \lesssim 500$ GeV[13, 14, 15]. We will return to this issue below.

We recognize the inherent subjectivity of the notion of naturalness. However, in order to decide whether one model is more natural than another, we need to introduce a measure of fine-tuning. As we discuss in the next section, this is traditionally done by checking the sensitivity of M_Z^2 rather than the Higgs mass as in Eq. (1), to the model parameters. Since both gauge and Higgs boson masses arise dynamically from the scalar potential, the corresponding sensitivities are not unrelated.

In the next section, we compare and contrast three different sensitivity measures, Δ_{EW}^{-1} , Δ_{HS}^{-1} and Δ_{BG}^{-1} that have been the subject of discussion in the literature. While much of what we say here and in the rest of this paper is a review, our perspective differs from that of other authors. We emphasize that while not a fine-tuning measure, Δ_{EW} (which is essentially determined by the particle spectrum) is nonetheless a very useful quantity because Δ_{EW}^{-1} quantifies the minimum fine-tuning present in any theory with a specific spectrum. We also highlight the importance of incorporating correlations between various parameters in discussions of fine-tuning, something ignored in many generic analyses. In Sec. 3, we provide simple examples that suggest that if we can find HS theories with specific correlations amongst parameters, the value of the traditional fine-tuning measure Δ_{BG}^{-1} would automatically be close to Δ_{EW}^{-1} . We then set up the radiatively driven natural SUSY framework that we advocate for phenomenological analyses of natural models of SUSY, and review its phenomenological implications in Sec. 5. We conclude with our perspective and a brief summary in Sec. 6.

2 Quantifying fine-tuning

The inherent subjectivity of the notion of fine-tuning is reflected in the fact that there is no universally accepted criterion for when a theory is fine-tuned. Everyone agrees that a model is natural if its predictions can be obtained without the need for large cancellations between various *independent* contributions that are combined to obtain the predicted value of any quantity: see *e.g.* our discussion following (1). As we will see below, the differences between various fine-tuning measures originate in whether all truly independent contributions are really included, and (to a lesser degree) on the sensitivity measure used. Our purpose is to address whether supersymmetric models can be natural in light of what we have learnt from LHC8 data. Furthermore, we will limit our discussion to the Minimal Supersymmetric

Standard Model (MSSM) since part of our goal is to examine whether naturalness considerations unequivocally force us to consider extended frameworks.

With this in mind, we discuss three different fine-tuning measures that have received attention in the recent literature. As we have noted above, the predicted value of M_Z^2 obtained from the minimization of the one-loop-corrected Higgs boson potential

$$\frac{M_Z^2}{2} = \frac{m_{H_d}^2 + \Sigma_d^d - (m_{H_u}^2 + \Sigma_u^u) \tan^2 \beta}{\tan^2 \beta - 1} - \mu^2, \quad (2)$$

is the starting point for most discussions of fine-tuning [17, 18, 19]. This expression is obtained using the weak scale MSSM Higgs potential and all parameters in Eq. (2) are evaluated at the scale $Q = M_{SUSY}$. The Σ s in Eq. (2), which arise from one loop corrections to the Higgs potential, are the analogue of the C_3 term in (1). Explicit forms for the Σ_u^u and Σ_d^d are given in the Appendix of Ref. [20].

2.1 Δ_{EW}

Requiring that the observed value of M_Z^2 is obtained without large cancellations means that each of the various terms on the right-hand-side of Eq. (2) has to be comparable to M_Z^2 in magnitude. Thus the fine tuning in Eq. (2) can be quantified by Δ_{EW}^{-1} , where [21, 22, 20]

$$\Delta_{EW} \equiv \max_i |C_i| / (M_Z^2/2). \quad (3)$$

Here, $C_{H_d} = m_{H_d}^2/(\tan^2 \beta - 1)$, $C_{H_u} = -m_{H_u}^2 \tan^2 \beta/(\tan^2 \beta - 1)$ and $C_\mu = -\mu^2$. Also, $C_{\Sigma_u^u(k)} = -\Sigma_u^u(k) \tan^2 \beta/(\tan^2 \beta - 1)$ and $C_{\Sigma_d^d(k)} = \Sigma_d^d(k)/(\tan^2 \beta - 1)$, where k labels the various loop contributions included in Eq. (2). We immediately see that any upper bound on Δ_{EW} that we impose from naturalness considerations necessarily implies a corresponding limit on μ^2 . Thus higgsino masses are necessarily bounded from above in any theory with small values of Δ_{EW} .²

Before proceeding further, we remark that Δ_{EW} as defined here entails only weak scale parameters (see also Ref. [24]) and so has no information about the $\log \Lambda$ terms that cause weak scale physics to exhibit logarithmic sensitivity to HS physics as discussed in Sec. 1. For this reason Δ_{EW} is *not a fine-tuning measure* in the underlying HS theory, as already noted in Ref. [20]. It is nonetheless very useful because, as noted below, Δ_{EW}^{-1} yields a *lower bound* on the fine-tuning in any HS theory with a given SUSY spectrum. Moreover, we will see in Sec. 3 that this bound can be saturated in an appropriate HS theory with the same spectrum. For now we turn our attention to Δ_{HS} which includes the information of the large logarithms in its definition.

2.2 Δ_{HS}

The large logarithms that we have been discussing remain hidden in Eq. (2) because we have written this condition in terms of the parameters of the theory renormalized at the weak scale.

²This reasoning fails if the dominant contribution to the higgsino mass arises from SUSY breaking [23] and not from μ . If there are no singlets that couple to the higgsinos, such a contribution would be soft. However, in all HS models that we are aware of, the higgsino masses have a supersymmetric origin.

To make these explicit, we rewrite the *weak scale* parameters $m_{H_{u,d}}^2$ and μ that appear in Eq. (2) in terms of the parameters of the HS theory as,

$$m_{H_{u,d}}^2 = m_{H_{u,d}}^2(\Lambda) + \delta m_{H_{u,d}}^2; \quad \mu^2 = \mu^2(\Lambda) + \delta\mu^2, \quad (4)$$

where $m_{H_{u,d}}^2(\Lambda)$ and $\mu^2(\Lambda)$ are the corresponding parameters renormalized at the high scale Λ . In terms of the high scale parameters, the minimization condition now takes the form,

$$\frac{M_Z^2}{2} = \frac{(m_{H_d}^2(\Lambda) + \delta m_{H_d}^2 + \Sigma_d^d) - (m_{H_u}^2(\Lambda) + \delta m_{H_u}^2 + \Sigma_u^u) \tan^2 \beta}{\tan^2 \beta - 1} - (\mu^2(\Lambda) + \delta\mu^2). \quad (5)$$

The $\delta m_{H_{u,d}}^2$ and $\delta\mu^2$ terms contain the $\log \frac{\Lambda^2}{m_{\text{low}}^2}$ factors that appears in the C_2 term in (1). Various authors [13, 15, 14] have argued that this leads to rather stringent upper bounds on sparticle – most notably top squark – masses from naturalness considerations. We will see below that natural models of SUSY with top squarks beyond the reach of the LHC are perfectly possible.

We can now define a fine-tuning measure that encodes the information about the high scale origin of the parameters in a manner analogous to the definition of Δ_{EW} above by now requiring [22] that none of the terms on the right-hand-side of Eq. (5) are much larger than M_Z^2 . The high scale fine-tuning measure Δ_{HS} is thus defined to be

$$\Delta_{\text{HS}} \equiv \max_i |B_i| / (M_Z^2/2), \quad (6)$$

with $B_{H_d} \equiv m_{H_d}^2(\Lambda)/(\tan^2 \beta - 1)$, $B_{\delta m_{H_d}^2} \equiv \delta m_{H_d}^2/(\tan^2 \beta - 1)$, *etc.*

In models such as mSUGRA [16], whose domain of validity extends to very high scales, because of the large logarithms one would expect that the $B_{\delta m_{H_u}^2}$ contributions to Δ_{HS} would typically be much larger than any contributions to Δ_{EW} . The reason is that the term $m_{H_u}^2$ evolves from large m_0^2 through zero to negative values in order to radiatively break electroweak symmetry. Put differently, the loop terms $\delta m_{H_{u,d}}^2$ in Eq. (5) are typically much larger than the loop terms in Eq. (2) because of the presence of the large logarithm, and we typically have,

$$\Delta_{\text{HS}} \gg \Delta_{\text{EW}}. \quad (7)$$

Large cancellations between, for instance, $m_{H_u}^2(\Lambda)$ and $\delta m_{H_u}^2$ that result in small Δ_{EW} will nonetheless yield a large value of Δ_{HS} .

Before closing this section, we note a potential pitfall of using Δ_{HS} as a measure of fine-tuning. Although Δ_{HS} is a sensible measure of fine-tuning in a *generic* HS theory in that it captures effects of the HS origin of the underlying model parameters, it does not take into account the fact that in models with a small number of parameters, the various terms on the right-hand-side of (5) could be correlated. In this case, there could be automatic cancellations between the various terms that Δ_{HS} does not incorporate. In models where such cancellations occur automatically,³ using Δ_{HS} could erroneously lead us to infer that the model is fine-tuned. The possibility that correlations among parameters can lead to reduced fine-tuning has been mentioned in Ref. [28, 19, 29]. For another possibility, see Ref. [30].

³Cancellations between $m_{H_u}^2(\Lambda)$ and $\delta m_{H_u}^2$ are guaranteed for specially chosen values of $m_{H_u}^2(\Lambda)$ in the NUHM2 model. The HB/FP region of mSUGRA [25] and its generalizations [26], and the mixed-modulus-anomaly-mediated SUSY breaking model (also referred to as mirage mediation models) [27] provide other examples of such (partial) cancellations.

2.3 Δ_{BG}

The correlations that could be the pitfall of Δ_{HS} as a fine-tuning measure are most easily implemented in the traditional fine-tuning measure Δ_{BG} [17, 18, 31], defined as the fractional change in the output value of M_Z^2 given by (2) relative to the corresponding change in the input parameters,

$$\Delta_{\text{BG}} = \max_i |c_i| \equiv \max_i \left| \frac{a_i}{M_Z^2} \frac{\partial M_Z^2}{\partial a_i} \right|. \quad (8)$$

Here, the a_i 's are the underlying parameters of the theory. These would be the weak scale parameter set in the case of the pMSSM, in which case Δ_{BG} would be close to Δ_{EW} , or the HS parameter set for models such as mSUGRA. We would expect that in the latter case, aside from the possibility of correlations discussed in the previous paragraph, Δ_{BG} and Δ_{HS} will be very strongly correlated.⁴ Remember, however, that for the evaluation of Δ_{BG} , we need to combine terms on the right-hand-side of Eq. (5) to calculate the sensitivity coefficients c_i , above. If this combination results in cancellations because of underlying correlations between HS parameters of the theory the corresponding c_i , and concomitantly also Δ_{BG} , will automatically reduce, whereas Δ_{HS} (which does not know about these cancellations) remains unchanged. We thus generically expect that,⁵

$$\Delta_{\text{EW}} \leq \Delta_{\text{BG}} \lesssim \Delta_{\text{HS}}. \quad (9)$$

To make explicit what cancellations we are referring to, we see that for moderate to large values of $\tan \beta$, we can write Eq. (2), to a good approximation, as

$$\frac{1}{2}M_Z^2 \simeq -m_{H_u}^2 - \mu^2.$$

The weak scale values of $m_{H_u}^2$ and μ^2 that appear above can be written in terms of the HS parameters using the semi-analytic solutions to the one-loop renormalization group equations [32]. For instance, for $\tan \beta = 10$, we have [33, 34, 35],

$$\begin{aligned} -2\mu^2(m_{\text{weak}}) &= -2.18\mu^2, \\ -2m_{H_u}^2(m_{\text{weak}}) &= 3.84M_3^2 + 0.32M_3M_2 + 0.047M_1M_3 - 0.42M_2^2 \\ &\quad + 0.011M_2M_1 - 0.012M_1^2 - 0.65M_3A_t - 0.15M_2A_t \\ &\quad - 0.025M_1A_t + 0.22A_t^2 + 0.004M_3A_b \\ &\quad - 1.27m_{H_u}^2 - 0.053m_{H_d}^2 \\ &\quad + 0.73m_{Q_3}^2 + 0.57m_{U_3}^2 + 0.049m_{D_3}^2 - 0.052m_{L_3}^2 + 0.053m_{E_3}^2 \\ &\quad + 0.051m_{Q_2}^2 - 0.11m_{U_2}^2 + 0.051m_{D_2}^2 - 0.052m_{L_2}^2 + 0.053m_{E_2}^2 \\ &\quad + 0.051m_{Q_1}^2 - 0.11m_{U_1}^2 + 0.051m_{D_1}^2 - 0.052m_{L_1}^2 + 0.053m_{E_1}^2, \end{aligned} \quad (10)$$

⁴ Δ_{BG} would equal to Δ_{HS} if M_Z^2 depends linearly on the model parameters a_i , and radiative corrections embodied in Σ are ignored.

⁵This presumes that the dominant terms in M_Z^2 are (approximately) linear in the parameters a_i . The semi-analytic formulae in Eq. (11) below show that this is indeed the case, except for $a_i = m_{1/2}$. In the case that the sensitivity coefficient $c(m_{1/2})$ is the largest, Δ_{HS} can be twice as large as Δ_{BG} because M_Z^2 is quadratic in $m_{1/2}$.

where the parameters on the right-hand-side are evaluated at the GUT scale.⁶ For other values of $\tan\beta$, the functional form on the right-hand-side is the same except for somewhat different values of the coefficients. We can now use these to obtain the (approximate) sensitivity coefficients and hence Δ_{BG} , using the semi-analytic approximations in (11) above, assuming that the two-loop effects and the radiative correction terms Σ_u^u are small. We will return to the validity of these approximations in Sec. 3.

The sensitivity coefficients depend on the underlying parameters of the model. In the much-studied mSUGRA/CMSSM framework, the gaugino mass parameters all unify to a common parameter $m_{1/2}$ at the high scale (usually taken to be M_{GUT}) while *all* scalar masses are assumed to unify to m_0 , and the trilinear couplings to A_0 . In this case, because the HS model parameters are strongly correlated, Eq. (11) collapses to,

$$-2m_{H_u}^2(m_{\text{weak}}) = 3.78m_{1/2}^2 - 0.82A_0m_{1/2} + 0.22A_0^2 + 0.013m_0^2 \quad (\text{mSUGRA}).$$

We see that in the mSUGRA framework, the hallmark universality of scalar mass parameters accidentally leads to a tiny coefficient in front of m_0^2 . The smallness of this coefficient is an example of cancellations between $m_{H_u}^2(\Lambda)$ and $\delta m_{H_u}^2$ that occur, *e.g.* in the HB/FP region of mSUGRA *provided that contributions from $m_{1/2}$ and A_0 terms as well as from the radiative corrections Σ_u^u are also small*; see, however, Ref. [22]. In this parameter region, Δ_{BG} is significantly smaller than Δ_{HS} .

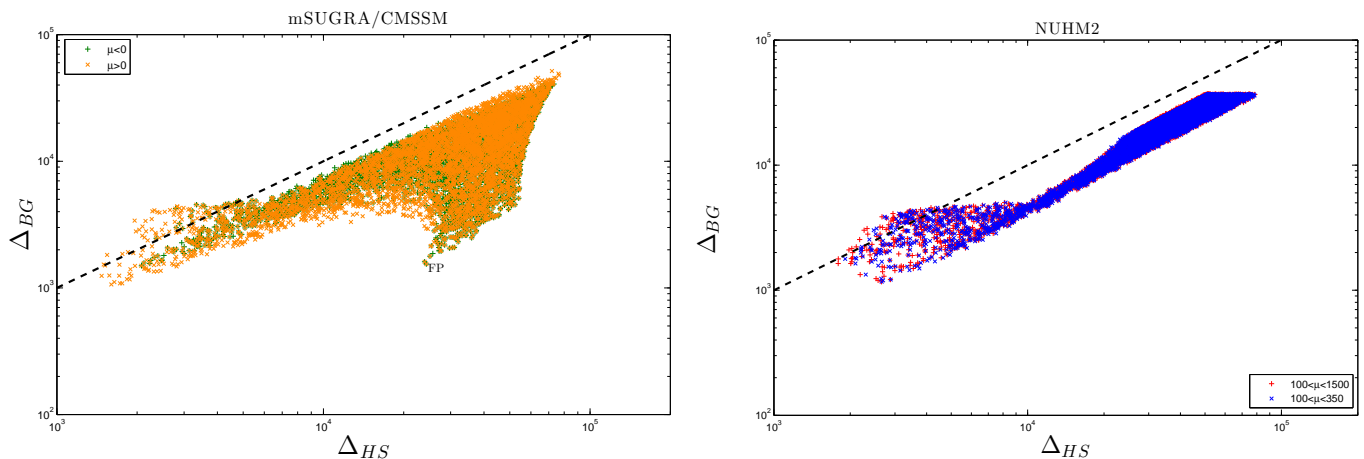


Figure 1: Plot of Δ_{BG} versus Δ_{HS} from scans of (a) the mSUGRA parameter space (left frame), and (b) the NUHM2 model parameter space (right frame), as detailed in Ref. [37].

In Fig. 1 we show Δ_{BG} vs. Δ_{HS} for a scan over the parameter space of phenomenologically consistent points in (a) the mSUGRA model, and (b) the parameter space of the non-universal

⁶Eq. (11) is written in the same convention as that used for input parameters into ISAJET. The same convention is used throughout this paper. We warn the reader that in the convention of [7], the signs of the $A_i M_j$ terms in (11) would have to be flipped.

Higgs mass model (NUHM2) which is just mSUGRA except that the GUT-scale Higgs mass parameters $m_{H_u}^2$ and $m_{H_d}^2$, or equivalently the weak scale values of μ and m_A , are chosen to be independent of the mSUGRA parameters [36]. For details of the scan, we refer the reader to Ref. [37] from which this figure has been adapted.

We see that for both models, Δ_{HS} and Δ_{BG} are strongly correlated as anticipated above, and that the inequality (9) is satisfied. The handful of points where $\Delta_{\text{HS}} < \Delta_{\text{BG}} < 2\Delta_{\text{HS}}$ are presumably for the cases where $c(m_{1/2})$ is the largest of the sensitivity coefficients. While Δ_{BG} is generally comparable to Δ_{HS} , there is a subset of points in the mSUGRA case (marked FP) where Δ_{BG} is substantially smaller than Δ_{HS} . This is the hyperbolic branch/focus point region [25] of mSUGRA where the correlations between the parameters significantly reduce the fine-tuning as discussed in the previous paragraph. We see nevertheless, that Δ_{BG} (as well as Δ_{HS} , of course) is always larger than $\sim 10^3$ so that both models would be considered fine-tuned to at least a part-per-mille, if Δ_{BG}^{-1} is used as the fine-tuning measure.

We contrast this with Δ_{EW} which was shown [22] to have a minimal value of $\mathcal{O}(100)$ in the mSUGRA model (after the imposition of LHC Higgs and sparticle mass constraints) but could be as small as 10 in *special regions* of the NUHM2 parameter space [20, 37]. Following our earlier discussion, we interpret this to imply that *any theory* that leads to an mSUGRA-like sparticle spectrum with only MSSM particles at the SUSY scale, will be fine-tuned to at least the percent level, but leaves open the possibility of finding a much less fine-tuned HS model that reduces to NUHM2 (with specific correlations between NUHM2 parameters) as the effective theory at a scale near $Q = M_{\text{GUT}}$. We will address this in Sec. 3.

2.4 The utility of Δ_{EW}

We have discussed three quantities, Δ_{EW} , Δ_{HS} and Δ_{BG} related to fine-tuning, that satisfy (9). Of these, Δ_{HS} and Δ_{BG} include information about potential enormous cancellations that may be needed if there are states with mass scales vastly greater than experimentally accessible energies that couple to SM particles and their superpartners. In contrast, Δ_{EW} completely disregards these cancellations since, by construction, it has no sensitivity to ultra-violet physics. The inequality (9) shows that Δ_{EW}^{-1} provides a bound on the fine-tuning measure in a generic quantum field theory in that measures the minimal fine-tuning that is present for a given spectrum.⁷ While a model with a small value of Δ_{EW} is not necessarily free of fine-tuning, a model with a large value of Δ_{EW} is always fine-tuned. Δ_{HS} and Δ_{BG} are usually comparable and differ from each other only when there are correlations between HS parameters that lead to automatic cancellations between terms on the right-hand-side of (5), or equivalently (1). Inclusion of these correlations is essential to obtain a true sense of fine-tuning in a particular model.

⁷We are well aware that the inequality $\Delta_{\text{EW}} \leq \Delta_{\text{BG}}$ need not hold in the strict mathematical sense. An extreme, albeit contrived, example may be a meta-theory where all mass parameters are determined by a single mass scale m , so that $M_Z^2 = am^2$, with a fixed by the theory. In this case $\Delta_{\text{BG}} = 1$, whereas Δ_{EW} , as we have defined it, may well be larger. (This is because our definition of Δ_{EW} , like that of Δ_{HS} , does not incorporate correlations between parameters.) In such a theory (if it exists) fine-tuning is a vacuous concept. Despite this, we believe Δ_{EW} provides a useful bound on the fine-tuning because it applies in all models where M_Z^2 receives sizeable contributions from two or more *uncorrelated* terms enhanced by $\log \Lambda$. This is indeed the case in many models.

The utility of Δ_{EW} arises from the fact that it is essentially determined by the weak scale spectrum [20], *i.e.*, different HS theories that lead to the same sparticle spectrum will yield nearly the same value of Δ_{EW} , even though these may have vastly different values of Δ_{HS} or Δ_{BG} . A small value of Δ_{EW} in, say, some region of parameter space of the NUHM2 model, offers the possibility that one may discover a HS theory with essentially the same spectrum that simultaneously has a small value of $\Delta_{\text{BG}} \sim \Delta_{\text{EW}}$. This HS model (if it exists) will then be the underlying theory with low fine-tuning. Since many broad features of the phenomenology are determined by the spectrum, much of the phenomenology of the (unknown) underlying theory is the same as those of the NUHM2 model with the same spectrum.⁸ The underlying philosophy behind much of our recent work [20, 38] is that the NUHM2 model acts as a surrogate for the yet-to-be discovered theory with low fine-tuning. The other side of the same coin is that if we discovered superpartners and found that these exhibited the spectrum of the mSUGRA model with $m_h = 125\text{--}126$ GeV, we would be forced to conclude that *any underlying theory* that led to this spectrum would have to be fine-tuned [22].

3 How correlations (nearly) reduce Δ_{BG} to Δ_{EW} : A simple example

We are led to conclude that Δ_{BG} which includes information of both UV physics and readily facilitates the inclusion of possible correlations among HS parameters that lead to *automatic cancellations* in (5) is the optimal measure of fine-tuning in quantum field theory. In contrast we have argued that Δ_{EW}^{-1} yields a useful bound on the fine-tuning for a given sparticle spectrum. In this section, we ask if the numerical value of Δ_{BG} would cause it to approach the value of Δ_{EW} once correlations among the HS parameters are incorporated. For our study, we adopt the NUHM2 model cases from Table 1 of Ref. [20] that resulted in low values of Δ_{EW} . Specifically, we have have:

CaseA : $m_0 = 2.5$ TeV, $m_{1/2} = 0.4$ TeV; $A_0 = -4$ TeV; $\tan\beta = 10$; $m_A = 1$ TeV; $\mu = 150$ GeV,

CaseB : $m_0 = 4$ TeV, $m_{1/2} = 1$ TeV; $A_0 = -6.4$ TeV; $\tan\beta = 15$; $m_A = 2$ TeV; $\mu = 150$ GeV.

Table 1 of Ref. [20] shows that a change of $\sim 1\%$ in the GUT scale values of $m_{H_u}^2$ caused Δ_{EW} to alter by $\sim 60\text{--}100\%$. This had led us to suggest that if there was an underlying meta-theory in which $m_{H_u}^2$ and m_0^2 were tightly correlated instead of being independent parameters as in the NUHM2 model, this underlying theory might not be fine-tuned.

In the NUHM2 model, Case A yields $\Delta_{\text{BG}} = 3168$ and $\Delta_{\text{EW}} = 11.3$, while for Case B we have $\Delta_{\text{BG}} = 8553$ and $\Delta_{\text{EW}} = 16.9$. We immediately see that since Δ_{EW} is two orders of magnitude smaller than Δ_{BG} , in order to check whether the correlations indeed reduce Δ_{BG} to (near) Δ_{EW} , the former would need to be computed to better than the percent level. This

⁸Exceptions to this, would be phenomenological aspects that are very sensitive to the mass correlations special to the NUHM2 model; since these correlations would depend on the details of the model, the NUHM2 model might not represent these faithfully. But many mass reaches at the LHC, or SUSY contributions to the anomalous magnetic moment of the muon or $b \rightarrow s\gamma$ (as long as there are no large cancellations between various SUSY contributions), or even dark matter phenomenology would be expected to be the same.

precludes the use of semi-analytic 1-loop expressions (11) that ignore two loop effects and also evaluate the coefficients for a fixed value of $\tan \beta$ for the computation of Δ_{BG} .⁹ We clearly need a different procedure to evaluate Δ_{BG} in the meta-theory in which various NUHM2 parameters are correlated; *i.e.*, the meta-theory has fewer independent parameters than contained in the NUHM2 parameter set.

We use the following multi-step procedure based on ISAJET[39] for a reliable evaluation of Δ_{BG} :

1. Since the sensitivity coefficients needed for the evaluation of Δ_{BG} depend on GUT-scale parameters, for the NUHM2 point of interest (for which ISAJET uses the weak scale values of μ and m_A as inputs), we first evaluate $m_{H_u}^2(\text{GUT})$ and $m_{H_d}^2(\text{GUT})$ using two-loop renormalization group evolution instead of the one-loop semi-analytic formulae mentioned above.
2. We have created a program that uses these GUT-scale values of Higgs parameters together with other GUT-scale SUSY parameters to iteratively evaluate the SUSY spectrum. For this code, $|\mu|$ and M_Z are outputs that (nearly) coincide with the input value of $|\mu|$ and the observed value of M_Z . We use the GUT-scale values of gauge and Yukawa couplings from the last iteration for this calculation. The values of Σ_u^u and Σ_d^d are also re-evaluated.
3. To evaluate the sensitivity coefficients that enter the computation of Δ_{BG} , we now incrementally change each of the *independent* GUT-scale input parameters one-by-one (keeping all other parameters fixed) and reevaluate M_Z^2 . The sensitivity coefficient is then obtained using $c_i = \frac{a_i}{M_Z^2} \frac{\delta M_Z^2}{\delta a_i}$. The largest of the sensitivity coefficients is taken as Δ_{BG} . Within the NUHM2 model, the parameters $m_0, m_{1/2}, A_0, m_{H_u}^2(\text{GUT})$ and $m_{H_d}^2(\text{GUT})$ are all independent, and so each one of these has a sensitivity coefficient that enters the evaluation of Δ_{BG} . As noted, this gives $\Delta_{\text{BG}} = 3168$ and $\Delta_{\text{BG}} = 8553$ for Cases A and B, respectively. The situation is quite different if the NUHM2 arises from a meta-theory in which the parameters are correlated as described below.
4. Next, motivated by our earlier studies, we imagine that the NUHM2 is derived from a meta-model in which A_0 is not an independent parameter but is fixed in terms of m_0 by $A_0 = \xi_A m_0$, with $\xi_A \sim -1.6$. This correlation reduces \tilde{t}_1 contributions to Σ_u^u and simultaneously raises m_h to its observed value [21]. In the meta-model, the sensitivity coefficient corresponding to A_0 should not be included during the evaluation of Δ_{BG} because A_0 is not an independent parameter. For the two cases that we examined (and likely over much of parameter space), the value of Δ_{BG} was fixed by sensitivity coefficients other than c_{A_0} , and so remains unchanged from its value in the NUHM2 model.
5. Recalling that the adjustment of the GUT-scale value of $m_{H_u}^2$ was key to obtaining a low value of Δ_{EW} in the NUHM2 framework [20], we assume that, like A_0 , $m_{H_u}^2(\text{GUT})$ is also not an independent parameter in the meta-theory. Since the sensitivity to $m_{H_u}^2(\text{GUT})$ was

⁹We emphasize that over much of the parameter space of the NUHM2 model, the evaluation of Δ_{BG} using (11) (as was done in Ref. [37]) will be reliable. Only when the cancellation between the various terms approach the percent level will this procedure become suspect.

Correlation	Case A	Case B
None	3168	8553
$A_0 = \xi_A m_0, m_{H_u}^2 = \xi_H m_0^2$	257	1123
$m_{1/2} = \xi_{1/2} m_0$	15.4	55
Δ_{EW}	11.3	17

Table 1: Values of Δ_{BG} for the two cases of the NUHM2 model introduced in the text. The first row shows the value of Δ_{BG} without any correlations; in the second row we take A_0 and $m_{H_u}^2(\text{GUT})$ to be determined by m_0 with $\xi_H = 1.64$ for Case A, and 1.70 for Case B, with $\xi_A = -1.6$ for both Cases. In the third row we assume that the value of $m_{1/2}$ is also determined by m_0 with $\xi_{1/2} = 0.16$ (0.25) for Case A (Case B). The last row shows the value of Δ_{EW} .

dominant in the NUHM2 model, viewing this as a dependent parameter can dramatically reduce Δ_{BG} . Taking $m_{H_u}^2(\text{GUT}) = \xi_H m_0^2$ with $\xi_H = 1.64$ (1.70) in Case A (Case B) reduces Δ_{BG} by about an order of magnitude.

- Finally, if we assume that the gaugino masses are also not independent but given by $m_{1/2} = \xi_{1/2} m_0$ with $\xi_{1/2} = 0.16$ (0.25) in Case A (Case B), Δ_{BG} drops by another order of magnitude. We emphasize that the spectrum and, in fact, all phenomenological predictions of this meta-theory will be identical to those of the NUHM2 model with the same parameters.

The impact of these correlations between the parameters of the meta-theory on Δ_{BG} is illustrated in Table 1. We see that in a meta-model with $\xi_A = -1.6$ which automatically reduces the value of Σ_u^u , correlating the GUT-scale parameter $m_{H_u}^2$ reduces Δ_{BG} by an order of magnitude. A reduction by another order of magnitude, leading to Δ_{BG} not far from Δ_{EW} , is obtained by also correlating $m_{1/2}$. The following remarks are worth noting.

- It is clear that the small values of Δ_{BG} in the penultimate row of Table 1 are the result of very substantial cancellations between various contributions. This makes its evaluation numerically delicate. Here, we have chosen the values of ξ_H and $\xi_{1/2}$ directly from Ref. [20] without attempting to check whether adjusting these will bring Δ_{BG} yet closer to Δ_{EW} . Indeed, with our present code, we are unable to tell whether the inequality $\Delta_{BG} \geq \Delta_{EW}$ is saturated within numerical error. The main new result is that, as anticipated, correlations among high scale parameters substantially reduce Δ_{BG} , and if we are able to find a meta-theory that results in these correlations, this theory will have low fine-tuning.
- The reader may be bothered by the fact that ξ_H and $\xi_{1/2}$ change somewhat from Case A to Case B. However, this is not an issue since it is entirely possible that A_0 , $m_{H_u}^2(\text{GUT})$ and $m_{1/2}$ are not correlated to just m_0 in the meta-theory; *i.e.*, the ξ_\bullet could well be functions also of other parameters. The fact that the ξ_\bullet are not widely different between cases, and are $\mathcal{O}(1)$ perhaps lends some support for our picture.

To recap, there are special regions of the parameter space of the NUHM2 model with small values of $\Delta_{EW} \sim 10 - 20$. In these regions, the value of Δ_{HS} , or even Δ_{BG} with all NUHM2

parameters treated independently, is large $\sim 10^3$. However, if we assume that the NUHM2 model is derived as the effective theory with (some of) its correlated as described above, we find that the value of Δ_{BG} drops dramatically and assumes values not far from Δ_{EW} . The parent theory, if it exists, that gives rise to these correlations among NUHM2 model parameters will have much lower fine-tuning than in the NUHM2 model. We freely admit that we do not have any idea of how the required correlations between parameters will arise – this will surely require a complete understanding of how supersymmetry is broken and how this breaking is communicated to MSSM superpartners – or even whether what we are suggesting is possible. Our point is that we can consistently speculate about such a possibility only in models where Δ_{EW} is small. Since many aspects of the phenomenology are fixed only by the super-partner spectrum, we can regard the NUHM2 framework with low value of Δ_{EW} as a surrogate for the underlying (unknown) meta-theory with low fine-tuning, and examine the experimental implications at the LHC within this framework. This forms the subject of the next section.

4 Radiatively Driven Natural Supersymmetry (RNS)

It is clear from (3) that a low value of μ^2/M_Z^2 is a necessary (though not sufficient) condition for obtaining a small value of Δ_{EW} . Since, aside from radiative corrections, (2) reduces to

$$\frac{1}{2}M_Z^2 \simeq -m_{H_u}^2 - \mu^2,$$

for moderate to large $\tan\beta$ it is clear that a weak scale value of $m_{H_u}^2$ close to M_Z^2 guarantees a correspondingly small value of μ^2 . This can always be realized in the NUHM2 framework since $m_{H_u}^2(\text{GUT})$ is an adjustable parameter. From the perspective of the NUHM2 framework this may necessitate a fine-tuning. However, as discussed a length in the last section, it leaves open the possibility of finding a HS theory with essentially the same mass spectrum that is fine-tuned at the level of Δ_{EW}^{-1} , not Δ_{BG}^{-1} as computed in the NUHM2 model.

To find these low Δ_{EW} solutions, we perform scans of the NUHM2 parameter space as described in detail in Ref. [20, 38, 37], requiring that (1) electroweak symmetry is radiatively broken, (2) LEP2 and LHC bounds on superpartner masses are respected, and (3) that the value of m_h is consistent with the value of the Higgs boson mass measured at the LHC. The low Δ_{EW} solutions of course have low values of $|\mu|$, and generally have $A_0 \sim -(1-2)m_0$; this value typically leads to a cancellation of the \tilde{t}_1 contribution to Σ_u^u (the \tilde{t}_2 contribution is suppressed if $m_{\tilde{t}_2} \sim (2.5-3)m_{\tilde{t}_1}$), and at the same time leads to large intra-generational top squark mixing that is required to raise the Higgs mass to ~ 125 GeV. Since the required small value of $|\mu|$ is obtained by $m_{H_u}^2$ being driven from its GUT scale choice to close to $-M_Z^2$ at the weak scale, this scenario has been referred to as *Radiatively Driven Natural Supersymmetry* (RNS). It can be used a surrogate for an underlying natural model of supersymmetry, and we urge its use for phenomenological analysis.

The RNS spectrum is characterized by:

- the presence of four higgsino-like states \tilde{Z}_1, \tilde{Z}_2 and \tilde{W}_1^\pm with masses in the 100-300 GeV range, and mass splitting $\sim 10-30$ GeV between \tilde{Z}_2 and the lightest supersymmetric particle (LSP);

- $m_{\tilde{g}} \sim 1.5 - 5$ TeV, with $\tilde{Z}_{3,4}$ and \tilde{W}_2^\pm masses fixed by (the assumed) gaugino mass unification condition;
- $m_{\tilde{t}_1} = 1 - 2$ TeV, $m_{\tilde{t}_2}, m_{\tilde{b}_{1,2}} \sim 2 - 4$ TeV; this is in contrast to many other studies that suggest that the stops should be in the few hundred GeV range, and so likely be accessible at the LHC.
- first and second generation sfermions in the 10 TeV range; this is not required to get low Δ_{EW} , but compatible [40] with it. This choice ameliorates the SUSY flavour and CP problems [41], and also raises the proton lifetime [42].

5 Phenomenology

We have seen that 100-300 GeV charged and neutral higgsinos, with a mass gap of 10-30 GeV with the LSP, are the hallmark of scenarios with $\Delta_{\text{EW}} \lesssim 30$. In this section, we present an overview of how SUSY signals may be detected in such scenarios, highlighting those signatures that may point to the underlying low value of $|\mu|$.

5.1 LHC

Within the RNS framework, light higgsinos are likely to be the most copiously produced superpartners at the LHC [38]. This is illustrated in Fig. 2 where we show various -ino production cross sections (squarks and sleptons are assumed to be heavy as we adopt the decoupling solution to the SUSY flavour problem) at LHC14. The small energy release in their decay makes their signals difficult to detect over SM backgrounds and we are led to investigate other channels for discovery of SUSY.

Gluinos: Gluino pair production leads to the usual cascade decay signatures in the well-studied multi-jet + multilepton channels. The fact that lighter charginos and neutralinos are higgsino-like rather than gaugino-like would affect the relative rates for topologies with specific lepton multiplicity, but are unlikely to significantly alter the reach which is mostly determined by the gluino production cross-section (which is essentially determined by the gluino and first-generation squark masses). A study of the gluino reach within the RNS framework shows that experiments at LHC14 should be sensitive to $m_{\tilde{g}}$ values up to 1700 GeV (1900 GeV), assuming an integrated luminosity of 300 (1000) fb^{-1} . It may also be possible to extract the value of $m_{\tilde{Z}_2} - m_{\tilde{Z}_1}$ from the end-point of the mass distribution of opposite sign/same flavour dileptons from the leptonic decays of \tilde{Z}_2 produced in gluino decay cascades, if the mass $\tilde{Z}_2 - \tilde{Z}_1$ mass gap is large enough [38]. We note, however, that experiments at the LHC can discover gluinos only over part of the range allowed by naturalness considerations.

Same Sign Dibosons: If $m_{1/2}$ happens to be small enough so that the bino and wino mass parameters are not hierarchically larger than $|\mu|$, the two charginos and all four neutralinos will be mixed gaugino-higgsino states with substantial mass gaps between the heavier-inos and the LSP. Moreover, these states will all be kinematically accessible at the LHC via electroweak production processes, and we will be awash in multilepton signals with hadronic activity only

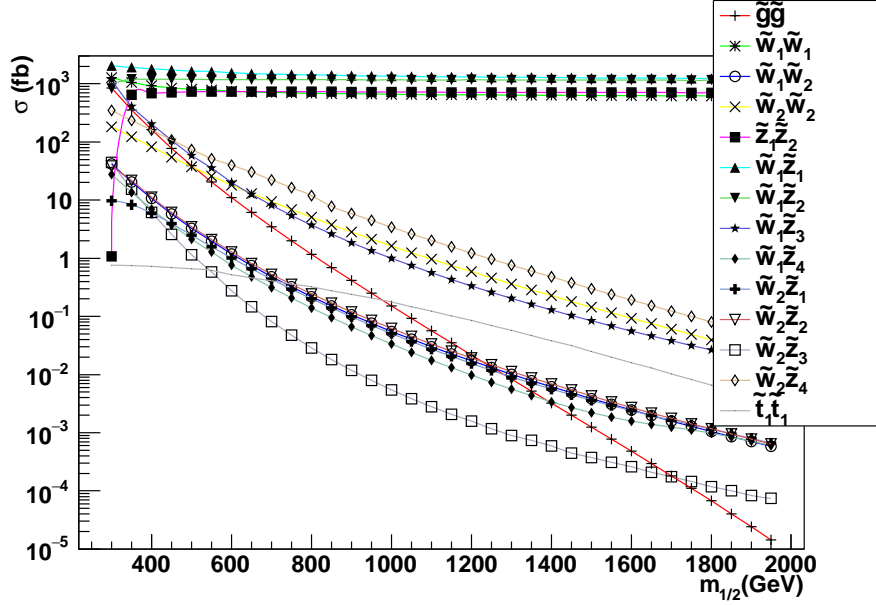


Figure 2: Plot of various NLO sparticle pair production cross sections versus $m_{1/2}$ along the RNS model line (12) for pp collisions at $\sqrt{s} = 14$ TeV.

from QCD radiation. In this fortuitous circumstance, the gluino signal discussed above will also likely be detectable.

The more typical scenario is when $|\mu| \ll M_{1,2}$ so that \tilde{W}_1 and \tilde{Z}_2 are higgsino-like and only 10-30 GeV heavier than \tilde{Z}_1 , \tilde{Z}_3 is dominantly a bino, and \tilde{W}_2 and \tilde{Z}_4 are winos. Because squarks are heavy, and the bino does not have couplings to W and Z bosons, electroweak production of \tilde{Z}_3 is dynamically suppressed. However, winos have large “iso-vector” couplings to the vector bosons so that wino cross sections can be substantial. Indeed we see from Fig. 2 that $\tilde{W}_2^\pm \tilde{W}_2^\mp$ and $\tilde{W}_2 \tilde{Z}_4$ cross sections remain substantial for high values of $m_{1/2}$.¹⁰ The large wino production cross-section leads to a novel signature involving same-sign dibosons produced via the processes $pp \rightarrow \tilde{W}_2^\pm (\rightarrow W^\pm \tilde{Z}_{1,2}) + \tilde{Z}_4 (\rightarrow W^\pm \tilde{W}_1^\mp)$. The decay products of the lighter chargino/neutralinos tend to be soft, so that the signal of interest is a pair of same sign high p_T leptons from the decays of the W -bosons, with limited jet activity in the event. This latter feature serves to distinguish this source from same sign dilepton events that might arise at the LHC from gluino pair production. We mention that $pp \rightarrow \tilde{W}_2^\pm \tilde{W}_2^\mp$ production (where one chargino decays to W and the other to a Z) also makes a non-negligible contribution to the $\ell^\pm \ell^\pm + E_T^{\text{miss}}$ channel when the third lepton fails to be detected. We emphasize here that this signal is a hallmark of all low μ models, if wino pair production occurs at substantial rates at the LHC.

We refer the reader interested in the details of the analysis required to separate the signal from SM backgrounds to Sec. 5 of Ref. [38]. We only mention that a hard E_T^{miss} cut and, very

¹⁰The $\tilde{W}_1 \tilde{Z}_3$ cross section is also significant, but falls more steeply with $m_{1/2}$ because the gaugino-higgsino mixing becomes increasingly suppressed.

NUHM2: $m_0=5$ TeV, $A_0=-1.6m_0$, $\tan\beta=15$, $\mu=150$ GeV, $m_A=1$ TeV

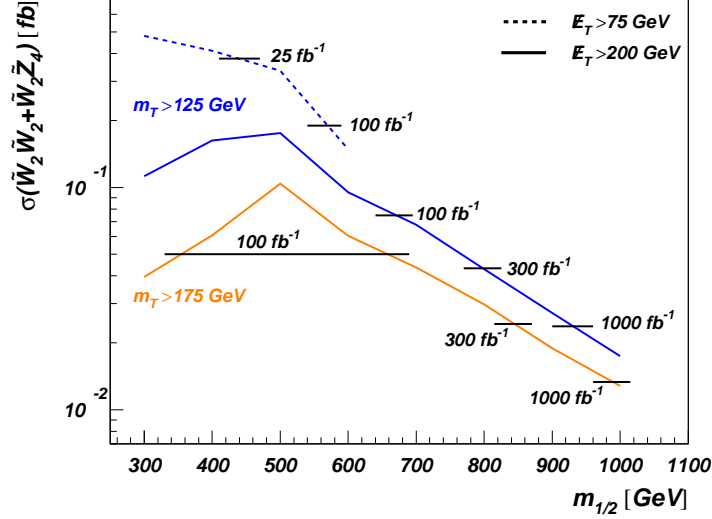


Figure 3: Same-sign dilepton cross sections (in fb) at LHC14 after cuts vs. $m_{1/2}$ along the RNS model line (12) from $\widetilde{W}_2^\pm \widetilde{Z}_4$ and $\widetilde{W}_2^\pm \widetilde{W}_2^\mp$ production and calculated reach for 100, 300 and 1000 fb^{-1} . The upper solid and dashed (blue) curves requires $m_T(\min) > 125$ GeV while the lower solid (orange) curve requires $m_T(\min) > 175$ GeV. The signal is observable above the horizontal lines.

importantly, a cut on

$$m_T^{\min} \equiv \min [m_T(\ell_1, E_T^{\text{miss}}), m_T(\ell_2, E_T^{\text{miss}})]$$

are very effective for suppressing the backgrounds relative to the signal. The 5σ reach of the LHC for an NUHM2 model line with,

$$m_0 = 5 \text{ TeV}, A_0 = -1.6m_0, \tan \beta = 15, \mu = 150 \text{ GeV}, m_A = 1 \text{ TeV}, \quad (12)$$

chosen to lead to low Δ_{EW} , is illustrated in Fig. 3 as a function of the gaugino mass parameter $m_{1/2}$. We show results for relatively soft cuts (dashed lines) and hard cuts on E_T^{miss} and m_T^{\min} . We see that with 300 fb^{-1} of integrated luminosity, experiments at the LHC will probe $m_{1/2}$ values up to 840 GeV, well in excess of what can be probed via cascade decays of gluinos.

Hard Trilepton Signals: Since low $|\mu|$ models yield such a large reach for winos, it is natural to ask how far the wino reach extends in the canonical trilepton channel, *i.e.*, from the reaction $pp \rightarrow \widetilde{W}_2 + \widetilde{Z}_4 X \rightarrow W + Z + E_T^{\text{miss}} + X$, long considered to be the golden mode for SUSY searches [43]. Here the E_T^{miss} arises from the $\widetilde{W}_1/\widetilde{Z}_{1,2}$ (whose visible decay products are very soft) daughters of the winos. A detailed analysis [38] shows that the reach via this channel extends to $m_{1/2} = 500$ (630) GeV for an integrated luminosity of 300 (1000) fb^{-1} , considerably lower than via the SSdB channel.

Four Lepton Signals: Low $|\mu|$ models, however, offer the possibility of $ZZ + E_T^{\text{miss}}$ events from $\widetilde{W}_2^+ \widetilde{W}_2^-$ or $\widetilde{W}_2^\pm \widetilde{Z}_4$ production, when both winos decay to Z plus a light chargino/neutralino. This leads to the possibility of a 4 lepton signal at LHC14. The reach in this channel was also mapped out in Ref. [38], by requiring 4 isolated leptons with $p_T(\ell) > 10$ GeV, a b -jet veto (to reduce backgrounds from top quarks), and $E_T^{\text{miss}} > E_T^{\text{miss}}(\text{cut})$. The value of $E_T^{\text{miss}}(\text{cut})$ was chosen so as to optimize the signal relative to SM backgrounds from ZZ , $t\bar{t}Z$, ZWW , ZZW , ZZZ and $Zh(\rightarrow WW^*)$ production. Since the background also includes a Z boson, and also because one of the four leptons in the signal occasionally arises as a leptonic daughter of the lighter \widetilde{W}_1 or \widetilde{Z}_2 , requiring a lepton pair to reconstruct M_Z , in fact, reduces the signal significance. It was found that in low $|\mu|$ models, the LHC14 reach via the 4ℓ search extends somewhat beyond that in the trilepton channel. Indeed a signal in this channel together with the SSdB signal could point to a SUSY scenario with small value of $|\mu|$ and a comparatively larger wino mass, as might be expected in RNS.

Soft Trileptons: The reader will remember from Fig. 2 that higgsino pair production is the dominant particle production mechanism at the LHC. This naturally leads to the question whether the $e\mu\mu$ signal from $\widetilde{W}_1 \widetilde{Z}_2$ might be observable, since the CMS and ATLAS experiments may be able to detect muons with $p_T(\mu)$ as small as 5 GeV. With this in mind, we examined the shape of the mass distribution of dimuons in the reaction $pp \rightarrow \widetilde{W}_1(\rightarrow e\nu\widetilde{Z}_1) + \widetilde{Z}_2(\rightarrow \mu^+\mu^-\widetilde{Z}_1)$ in Ref. [38], with cuts chosen to enhance the soft trilepton signal over large SM backgrounds. The signal dimuons would all have a mass smaller than the kinematic end point at $m_{\widetilde{Z}_2} - m_{\widetilde{Z}_1}$, while the background distribution would be expected to be much broader. Indeed it was found that there should be an enhancement of this distribution at small values of $m(\mu^+\mu^-)$, so that a *shape analysis* may well reveal the signal if $m_{1/2} < 400 - 500$ GeV, for $\mu = 150$ GeV. For larger values of $\mu/m_{1/2}$ the mass gap is so small that the resulting spectral distortion is confined to just one or two low mass bins. We conclude that while the soft-trilepton signal is unlikely to be a discovery channel, it could serve to strikingly confirm a SUSY signal in the SSdB or multilepton channels, and most importantly, point to a small value of $|\mu|$ if the parameters are in a fortuitous mass range.

Mono-jet and Mono-photon Signals: Many authors have suggested that experiments at LHC14 may be able to identify the pair production of LSPs via high E_T mono-jet or mono-photon plus E_T^{miss} events, where the jet/photon results from QCD/QED radiation. Many of these studies have been performed using non-renormalizable contact operators for LSP production. This overestimates the rates for mono-jet/mono-photon production at high E_T especially in models such as RNS where s -channel Z exchange dominates LSP pair production [44]. A careful study of this signal for the case of the higgsino LSP, incorporating the correct matrix elements as given within the RNS framework, shows that the signal will be very difficult to extract above the SM backgrounds, unless these can be controlled at the better than the percent level [45]. This is largely because the jet/photon E_T distribution as well as the E_T^{miss} distribution has essentially the same shape for the signal and the background. Alternatively, detection might be possible if the soft daughter leptons from the decays of the higgsino-like \widetilde{W}_1 and \widetilde{Z}_2 can serve to reduce the background in events triggered by the hard jet and/or E_T^{miss} .¹¹

¹¹This has been examined in Ref. [46] where the authors suggest this is feasible, at least for a sizeable mass gap. There were no explicit studies for a mass gap down to ~ 10 GeV that would be possible in the RNS

Int. lum. (fb ⁻¹)	$\tilde{g}\tilde{g}$	SSdB	$WZ \rightarrow 3\ell$	4ℓ
10	1.4	—	—	—
100	1.6	1.6	—	~ 1.2
300	1.7	2.1	1.4	$\gtrsim 1.4$
1000	1.9	2.4	1.6	$\gtrsim 1.6$

Table 2: Reach of LHC14 for SUSY in terms of gluino mass, $m_{\tilde{g}}$ (TeV), assuming various integrated luminosity values along an RNS model line introduced in (12).

Table 2 summarizes the projected reach of LHC14 in terms of the gluino mass within the RNS framework that we advocate be used for phenomenological analyses of natural SUSY. We see that for an integrated luminosity in excess of $\sim 100 \text{ fb}^{-1}$ the greatest reach will be obtained via the SSdB channel if we assume gaugino mass unification. More importantly, the SSdB channel provides a novel way to search for a SUSY signal in *any natural model of supersymmetry* since, as we have emphasized, the μ parameter needs to be small. In this case, there may be striking confirmatory signals in the 4ℓ and soft-trilepton channels in addition to the much-discussed clean trilepton signal from wino pair production.

5.2 ILC

Because light higgsinos are $SU(2)$ doublets, they necessarily have sizeable couplings to the Z boson, and so should be copiously produced in e^+e^- colliders, unless their production is kinematically suppressed. Since small $|\mu|$ is necessary for naturalness, electron-positron linear colliders that are being envisioned for construction are the obvious facility for definitive searches for natural SUSY. The issue, of course, is whether in light of the small visible energy release in higgsino decays it is possible to pull out the higgsino signal above SM backgrounds.

Here, we report preliminary results from an on-going study [47] of higgsino signals at an electron-positron linear collider with a centre-of-mass energy of 250 GeV (ILC250) that is seriously being considered for construction in Japan. For this study, we have chosen the NUHM2 point with $m_0 = 7025 \text{ GeV}$, $m_{1/2} = 568 \text{ GeV}$, $\mu = 115 \text{ GeV}$ with $\tan\beta = 10$. This case has gluinos and squarks beyond the current LHC reach (though it should be possible to find gluino and even wino signals at LHC14), and has $m_{\tilde{W}_1} = 117.3 \text{ GeV}$, $m_{\tilde{Z}_2} = 124 \text{ GeV}$ and $m_{\tilde{Z}_1} = 102.7 \text{ GeV}$, with $\Delta_{\text{EW}} = 14$. We view this point (ILC1) as an “easy case study” because of the rather large mass gap.

Backgrounds from $2 \rightarrow 2$ production processes typically have visible energies near 250 GeV, except when neutrino daughters from the decay of produced parents take away a large energy. In contrast, the signal has a visible energy smaller than 50 GeV. Except for a small contribution from the tail of the $e^+e^- \rightarrow WW$ production, the $2 \rightarrow 2$ backgrounds are efficiently removed by a cut on E_{vis} . Much more relevant are backgrounds from “two-photon” processes, $e^+e^- \rightarrow e^+e^-f\bar{f}$ where the final state electrons and positrons carry off the bulk of the energy and are lost down the beam-pipe. However, except when $f = c, b, \tau$ these events are back-to-back in

scenario.

the transverse plane and have very low E_T^{miss} . After the additional cut $E_T^{\text{miss}} > 20$ GeV, the signal from $e^+e^- \rightarrow \widetilde{W}_1^+ \widetilde{W}_1^- \rightarrow q\bar{q}\widetilde{Z}_1 + \ell\nu\widetilde{Z}_1$ production is readily visible in the $2j + 1\ell$ channel with an integrated luminosity of 100 fb^{-1} , where jets and leptons are defined to have transverse energies bigger than 5 GeV. Beam polarization is not necessary for this.

The signal from neutralino production¹² via $e^+e^- \rightarrow \widetilde{Z}_1 \widetilde{Z}_2 \rightarrow \widetilde{Z}_1 \ell^+ \ell^- \widetilde{Z}_1$ is also detectable with additional cuts $E_T^{\text{miss}} > 15$ GeV, $\Delta\phi(\ell\ell) < \pi/2$, as described in Ref. [47]. For this study, 90% electron beam polarization is required. Notice that despite the small leptonic branching ratio for \widetilde{Z}_2 decay, the signal is best seen via the leptonic decay of \widetilde{Z}_2 . This is because hadronic decays of \widetilde{Z}_2 lead mostly to single jet event topologies.

Ref. [47] also examines a more challenging case, for a point along the model-line (12) with $m_{1/2} = 1.2$ TeV. This yields $m_{\widetilde{W}_1} \simeq m_{\widetilde{Z}_2} = 158$ GeV, and a mass gap with the neutralino of just ~ 10 GeV. This point is chosen because it has $\Delta_{\text{EW}} = 28.5$, close to what we consider the maximum for naturalness, and a mass gap that is near the minimum, consistent with naturalness considerations. For this case, gluinos and all squarks (and likely also winos) are beyond the LHC14 reach.

For the heavier \widetilde{W}_1 and \widetilde{Z}_2 mass for this case, we have performed a study taking $\sqrt{s} = 340$ GeV, just below the $t\bar{t}$ threshold. If the ILC is constructed, and its energy upgraded to study the top quark threshold, we expect that there will surely be an ILC run close to this energy. The smaller mass gap leads to events with even less visible energy than in the ILC1 case study just discussed. In this case, requiring $E_{\text{vis}} < 30$ GeV along with cuts on E_T^{miss} and various jet and lepton angles in the transverse plane suffices to make the background negligible, and render the signal observable at the 5σ level [47]. Indeed, since there may well be no visible signal at LHC14 in this difficult scenario, the ILC could well be a discovery machine for SUSY!

Although we have not performed a parameter space scan, the fact that the signal can be extracted even in this nearly maximally difficult RNS case strongly suggests that higgsino signal will be observable at an e^+e^- collider provided of course that the higgsinos are kinematically accessible and that electron beam polarization is available (for the neutralino signal). In fact, we are currently investigating the prospects for mass measurements.

5.3 Dark Matter

Since the LSP is likely higgsino-like in *all* models with natural supersymmetry, it will annihilate rapidly (via its large coupling to the Z boson, and also via t -channel higgsino exchange processes) in the early universe. As a result, in natural supersymmetry the measured cold dark matter density cannot arise solely from *thermally produced higgsinos* (remember that these are lighter than ~ 300 GeV) in standard Big Bang cosmology. Dark matter is thus likely to be multi-component. What is very interesting, however, is that because naturalness considerations also impose an upper bound on $m_{\tilde{g}}$ and corresponding limits on electroweak gaugino masses (via gaugino mass unification), *the thermal higgsino relic density cannot be arbitrarily small*. Indeed, within the RNS framework, $\Omega_{\widetilde{Z}_1} h^2$ must be between $\sim 0.004 - 0.03$, as shown by Baer, Barger and Mickleson [48]. This has important implications for DM detection ex-

¹²Pair production of identical higgsinos, $\widetilde{Z}_1 \widetilde{Z}_1$ or $\widetilde{Z}_2 \widetilde{Z}_2$ has a much smaller cross section as the coupling of the higgsinos to Z is dynamically suppressed [7].

periments. Specifically, ton-size direct detection experiments such as Xe-1Ton that probe the spin-independent nucleon LSP cross section at the $10^{-47} - 10^{-46}$ pb level will be sensitive to entire range of the expected higgsino fraction. Thus, the outcome of these experiments will have important ramifications for naturalness.¹³

6 Concluding Remarks

Naturalness is a measure of how sensitive low energy masses and couplings are to the dynamics at hierarchically separated energy scales, and so is an attribute of the underlying high scale theory. The dynamics of the SM shows us that the Higgs boson mass exhibits quadratic sensitive to masses of new, heavy particles, if these couple to the Higgs boson. This sensitivity is correspondingly reduced if these particles have very weak couplings to the Higgs sector, or couple only indirectly at the multi-loop level. In theories that incorporate weak scale supersymmetry, the quadratic sensitivity to the masses of particles at very high scales is reduced to logarithmic sensitivity. In all these considerations, we agree with most discussions of naturalness and fine-tuning in much of the literature.

Where we evidently differ from many authors is that we allow for the possibility that model parameters that appear independent from our low energy perspective may really correlated within the as yet undiscovered underlying theory. These correlations, as we have argued in Sec. 3, can easily change the fine-tuning measure by a couple of orders of magnitude: our toy illustrations show that a theory that appears to be fine-tuned at parts per ten thousand may actually be fine-tuned at the few percent level.¹⁴ Ignoring these parameter correlations is what leads to stringent limits on top squarks that are usually advertized as the hallmark of natural supersymmetry [13, 14, 15]. Indeed, the measures Δ_{HS} and Δ_{BG} defined in Sec. 2 both incorporate the sensitivity of M_Z^2 to the physics of new particles at the high scale. However, the effect of parameter correlations is most simply encoded into Δ_{BG} , but is technically difficult to incorporate into Δ_{HS} , because the coefficients B_i in (6) cannot easily be written in terms of the model parameters in a simple way.

Whether or not a theory is (or is not) natural clearly depends on how very heavy particles couple to weak scale particles. This is a question of dynamics, and so cannot be answered by just looking at the weak scale spectrum of the theory. For this reason, we cannot regard Δ_{EW}^{-1} (which is essentially fixed by the spectrum) introduced in (3) as a measure of fine-tuning in the theory, in sharp contrast to the considerations in Ref. [37]. Despite this, we agree with both Ref. [37] and [3] that fine-tuning considerations using the weak scale theory is very useful, albeit for different reasons from these authors. We find that Δ_{EW} is extremely useful because it serves

¹³We should remind the reader that there are the usual caveats to this conclusion. For instance if physics in the sector that makes up the remainder of the dark matter entails late decays that produce SM particles, the neutralino relic density today could be further diluted; see *e.g.* Ref. [49].

¹⁴The reader may object that if we allow the possibility of correlations, one may even argue that the Higgs mass parameter may not be fine-tuned in even the SM. While this is logically possible, we are not imaginative enough to see how a quadratic sensitivity to say the GUT scale would be reduced by many orders of magnitude to a sensitivity at the percent or parts per mille level by parameter correlations. Of course, a symmetry (*e.g.* SUSY) does just this, but more typically, symmetries are not preserved to yield cancellations with the required precision.

as a bound on Δ_{BG} , the true fine-tuning measure: see Eq. (9). Any model that leads to a large value of Δ_{EW} is certainly fine-tuned. A small value of Δ_{EW} in some region of model parameter space does not guarantee the model is not fine-tuned. However, it leaves open the possibility that parameter correlations required to zero in on this special part of parameter space will, one day, be obtained from a more fundamental underlying framework. Evaluation of Δ_{BG} with these parameter correlations incorporated, will then yield a value (close to) Δ_{EW} . However, until such time that we have such a theory, it is useful to examine the low Δ_{EW} regions of the parameter space of phenomenologically promising models because these serve as surrogates for an underlying theory with low fine-tuning, as explained at the end of Sec. 3.

The RNS framework which, by construction, has a low value of Δ_{EW} , provides an explicit realization of such a program. Since many phenomenological results are sensitive to just the spectrum, these can be abstracted from the RNS model. RNS phenomenology is discussed in Sec. 5. In Fig. 4, we show the $m_{1/2} - \mu$ plane of the NUHM2 model with large m_0 together with contours of Δ_{EW} . Above and to the right of the $\Delta_{\text{EW}} = 30$ contour, we regard the spectrum to

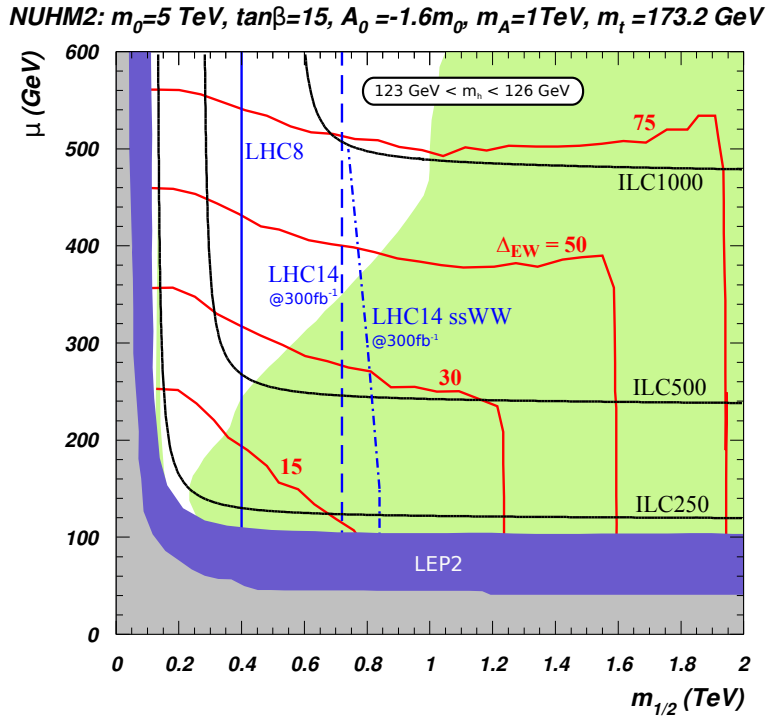


Figure 4: Plot of Δ_{EW} contours (red) labelled by the value of $\Delta_{\text{EW}} = 15, 30, 50$ and 75 in the $m_{1/2}$ vs. μ plane of NUHM2 model for $A_0 = -1.6m_0$ and $m_0 = 5$ TeV and $\tan\beta = 15$. We show the region accessible to LHC8 gluino pair searches (solid blue contour), and the region accessible to LHC14 searches with 300 fb^{-1} of integrated luminosity (dashed and dot-dashed contours). We also show the reach of various ILC machines for higgsino pair production (black contours). The very light-shaded (green) region has $\Omega_{\tilde{Z}_1}^{\text{std}} h^2 < 0.12$. The dark (light) shaded region along the axes is excluded by LEP2 (LEP1) searches for chargino pair production. To aid the reader, we note that $m_{\tilde{g}} \simeq 2.5m_{1/2}$.

be fine-tuned since the fine-tuning *must be* worse than $\Delta_{\text{EW}}^{-1} \sim 3\%$. The light-shaded (green) region is where the thermal higgsino relic density is smaller than its measured value, with the balance being made up by something else. The dashed line shows the LHC14 reach via the canonical search for gluinos, while the dot-dashed line shows our projection via searches in the novel SSdB channel discussed in Sec. 5.1. We see that LHC searches will, by themselves, not be able to cover the entire parameter space with $\Delta_{\text{EW}} < 30$. The remainder of this parameter space should be accessible, via a search for higgsinos at an e^+e^- collider operating at $\sqrt{s} = 600$ GeV.

To sum up, we stress that the fact that low scale physics is only logarithmically (and not quadratically) sensitive to the scale of ultra-violet physics remains a very attractive feature of softly broken SUSY models. The fact that it is possible to find phenomenologically viable models with low Δ_{EW} leads us to speculate that our understanding of UV physics is incomplete, and that there might be HS models with the necessary parameter correlations that will lead to comparably low values of the true fine-tuning parameter Δ_{BG} . The SUSY GUT paradigm remains very attractive despite the absence of new physics signals at LHC8. We hope that this situation will dramatically change with the upcoming run of the LHC.

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